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Quantitative methods and tools for studying political phenomena and processes

The application of mathematics to political processes, in particular, elections can be traced back to the times of the great French Revolution and even earlier. In the second half of the 20th century, the foundations of game theory were laid. The talk will be mostly concentrated around applications of game-theoretical methods to politics. We will consider the voting games, in particular, weighted games and their generalizations (vector-valued weighted voting games). Real-life examples will be considered. In particular, it will be demonstrated that the federal voting system in the USA is not weighted. The dual voting games to the given ones, i.e., the games in which blocking coalitions of the givem game are treated as winning, are considered, in particular, for the weighted games.

Also, power indices for voting games will be defined and calculated in some real-life cases. First of all we will consider the Shapley-Shubik power index. This index will be evaluated for parlament fractions. Also, the Shapley-Shubik power index allows us to explain why small parlament fractions can play significant role in the decision making.

We will define the Banzhaf power index for any voting game and compare it with the Shapley-Shubik power index. For the Banzhaf power index, there will be given an example that demonstrate paradoxal behavior of this index, namely that increasing of weight of a voter causes decreasing of his/her Banzhaf index. Analysis of voting in the Council of the European Union can be performed by means of the Banzhaf index.

Some other power indices (Deegan-Packel index, Johnson index) for voting games will be briefly discussed. More complicated voting systems with three or more outcomes will be considered. An analysis of the Condorcet paradox (paradox of voting). The examples of voting systems include plurality voting, (modified) Borda count, score voting, Copeland method, approval voting, Kemeny-Young voting, runoff voting etc. One of the most important results concerning the voting electoral systems is the Arrow Impossibility Theorem. In order to formulate this theorem some properties of the voting systems (Independence of Irrelevant Alternatives, Pareto efficiency etc.) should be introduced.

There are, however, some cases in which ther exists a Condorcet winner and it coincides with the plurality winner. In particular, this takes place in one-dimensional scale of players and ouncomes (one-dimensional political spactrum). In this case one can prove the median-voter theorem, which can be extended on considerably more general situations, namely that of single-peaked preferences. Note that this cannot be generalized over the two-dimensional spectrum as one can find a Condorcet paradox in this case.

A special attention will be paid to manipulability of voting systems and strategic voting. The manipulability property means that a player can win by reporting preferences other than his/her true preferences. In the case of two candidates the magority rule provides an example of a non-manipulable voting system. In the case of three or more candidates, the only non-manipulable voting system satisfying some additional natural conditions (e.g., Pareto condition) is the dictatorial voting system (Gibbard-Satterthwaite Theorem). It turns out that the nature of the Arrow Imposiibility Theorem is similar to that of the Gibbard-Satterthwaite Theorem, there exist even parallel proofs of both these results. There are numerous examples of strategic voting (also called a tactic voting) in real modern elections. One of the most known examples is that of 2011 parlamentary elections in Slovenia, in which about 30% of voters voted tactically, the largest quantity ever recorded.

We will also mention some geometric aspects of voting. For example, gerrymandering is a practice of changing the boundaries of electoral districts in order to obtain a political advantage, as the representation of supporters of a particular party or demographic group in the representative body can be artificially changed.

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